MULTI-BEAM RAINDROP SIZE DISTRIBUTION RETRIEVALS ON THE DOPPLER SPECTRA: INFLUENCE OF AVERAGING AND MEAN HORIZONTAL WIND CORRECTION

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ABSTRACT

Acquiring the raindrop size distribution (DSD) from radar data is still a challenge. For profiling radar, this distribution can be estimated from the Doppler spectra. However, the Doppler spectrum is not a direct measure of the DSD. The radial component of the wind shifts the Doppler spectrum related to the raindrop size distribution along the Doppler velocity interval. Furthermore, the Doppler spectrum may be broadened by turbulence effect. The Doppler spectra of rain are modelled using Rayleigh scattering, the gamma distribution for the raindrop size, a size-shape relationship and a Gaussian kernel for the turbulence. Comparing the measured Doppler spectrum with the modelled one, a non-linear least-square fit technique is employed to obtain the parameters of the raindrop size distribution ($D_0$, $\mu$) and the turbulence broadening factor of the raindrop size spectrum ($\sigma$). The intercept parameter ($N_w$) and the radial wind component ($v_0$) are estimated by scaling the broadened DSD along the spectral reflectivity axis and the Doppler velocity axis respectively. This approach has been proposed in [1]. So far, this technique has not been further developed, exploited and validated. It is our intention to investigate this methodology using the S-band Doppler-polarimetric TARA radar to retrieve raindrop size distribution from drizzle to heavy precipitation with high spatial and time resolution. Since TARA can profile in three directions, three raindrop size distribution profiles are estimated, which can give insight in the microphysical homogeneity of the precipitation. Because TARA can be used as wind profiler, the mean horizontal wind is estimated and a correction to remove the effect of the radial mean horizontal wind is implemented in the retrieval procedure. Comparison with and without correction is done. Another issue is averaging. Should the Doppler spectra be averaged before the non-linear least-square fit? How should they be averaged and how many averages should be carried out? These questions are discussed. High resolution multi-beams raindrop size distribution retrievals are illustrated and discussed.

1. MICROPHYSICAL MODEL OF RAINDROPS

The Doppler spectrum, or spectral reflectivity $sZ_{int}(\tilde{v})$, can be expressed as the Doppler spectrum resulting from the raindrop size distribution, $sZ_{int}(\tilde{v})$, convoluted by a Gaussian-shaped kernel of width $\sigma_0$ modelling spectral broadening. Spectral broadening has several causes, among them turbulence and wind shear in the radar resolution volume.

$$sZ_{int,mod}(\tilde{v}) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left(-\frac{(\tilde{v} - \tilde{v}_0)^2}{2\sigma_0^2}\right) sZ_{int}(\tilde{v}) d\tilde{v}$$

(1)

where $\tilde{v}$ and $\tilde{v}_0$ are Doppler velocities and

$$sZ_{int}(\tilde{v}) = N(D(\tilde{v})) \sigma_{int}(D(\tilde{v})) \frac{dD}{d\tilde{v}}$$

(2)

The radar cross section, $\sigma_{int}$, is calculated using Rayleigh-Gans scattering of spheroidal hydrometeors ([2], chap. 2). It strongly depends on the raindrop equivolume diameter, $D$. The raindrop size distribution, $N(D)$, follows the gamma distribution:

$$N(D) = N_e f(\mu) \left( \frac{D}{D_0} \right)^\mu \exp\left[-(3.67 + \mu) \frac{D}{D_0} \right]$$

(3)

and

$$f(\mu) = \frac{6}{(3.67)^{\mu+4}} \frac{\Gamma(\mu+4)}{\Gamma(\mu+4)}$$

(4)

where the triplet ($D_0$, $N_e$, $\mu$) characterize the distribution. $D_0$ is the median volume diameter and $\mu$ the shape parameter. The radial velocity is written as

$$\tilde{v}(D) = \left( \frac{\rho_0}{\rho} \right)^{1/4} \left[ 9.65 - 10.3 \exp(0.6D) \right] \sin \alpha + v_0$$

(5)

where $\rho_0$ and $\rho$ are the air densities at sea level and altitude of the radar resolution volume. The angle $\alpha$ is the elevation. Due to the contribution of the radial ambient wind, the Doppler spectrum experiences a shift of length $v_0$ along the Doppler velocity axis.

To calculate the radar cross section of the raindrops, Andsager shape-size relationship [3] is used for $1.1 \leq D \leq 4.4$ mm and Beard and Chuang relationship [4] is applied otherwise. Further, the probability density function of the raindrop orientation angle is described by Mardia axial distributions ([2], p. 68).

2. RETRIEVAL ALGORITHM

The retrieval algorithm obtains the three parameters of the DSD ($D_0$, $N_e$, $\mu$) and the dynamic parameters ($v_0$, $\sigma_0$) by fitting modelled spectra to measured spectra. An optimization procedure has to minimize the difference (or error) between the fitted spectrum and the measured spectrum by varying the five input parameters.
\[
\min \sum_{v}^{v_{\text{low}}} \left(10\log\left(sZ_{\text{mod}}(v)\right) - 10\log\left(sZ_{\text{mod}}(v, \Psi)\right)\right)^2
\]  

where \(\Psi\) contains all three raindrop size distribution parameters, the spectral broadening and the ambient wind velocity. The five parameter minimization problem (Eq. 6) can be simplified by separating the retrieval of the intercept parameter:

\[
sZ_{\text{mod}}(v) = sZ_{\text{mod}}(v, D_0, \mu, \sigma_0) N_w + \varepsilon
\]

This allows the derivation of estimates of the intercept parameters without nonlinear fitting. For this linear fit, the values of \(D_0\), \(\mu\) and \(\sigma_0\) have to be known and \(v_0\) is set to 0. A second simplification of Eq. 6 is done on the estimation of \(v_0\). Assuming the other four parameters of the model are known, the shift between the modelled and the measured spectrum can be obtained by determining the lag of the cross correlation of the measured and the modelled spectrum

\[
sZ_{\text{mod}}(v, D_0, \mu, 0, \sigma_0) N_w.
\]

The separate estimation of the intercept parameter and the ambient wind velocity results in a three parameter nonlinear least squares problem. The three parameters are \(D_0\), \(\mu\) and \(\sigma_0\). This approach is followed in \cite{1} and \cite{5}. Because of multiple minima in the cost functions, an iterative cascaded retrieval algorithm, like in \cite{5}, is preferred to obtain them.

### 3. ALGORITHM APPLICATION ON NON-AVERAGED DOPPLER SPECTRA

Because TARA has 3 beams for wind profiling, the retrieval technique can be applied on HH- and VV-spectra of the main beam (MB), which is dual-polarized, and on VV-spectra of the offset beams, OB1 and OB2, which are single-polarized. OB1 and OB2 are 15 deg. away from the main beam in two orthogonal directions. For the presented measurements, the elevations of MB, OB1 and OB2, are 75\(^\circ\), 90\(^\circ\) and 69\(^\circ\) respectively. Spectral polarimetric processing is performed to obtain noise- and clutter-free dealiased Doppler spectra \cite{6}-\cite{7} for the dual-polarized beam and classical spectral processing is carried out for the single-polarized beams. This full processing is implemented real-time for the TARA radar \cite{8}.

To strengthen the retrieval algorithm, two modifications have been implemented. The first one is the reduction of the search interval of \(\mu\), which is now \([-1, 5]\). The second one is the reduction of the search interval of \(D_0\) using the reflectivity value. Varying the drop size distribution parameters leads to simulated reflectivity values. After a large number of runs, an interval of median volume diameters can be associated to an interval of reflectivity values, assuming that \(N_w\) varies from \(10^{2.5}\) to \(10^{7}\). For example, if the measured reflectivity is between 0 and 10 dBZ, like in the example of Fig. 1, the search interval of \(D_0\) is \([0.3, 0.9]\) mm.

Note that retrieval results for the main beam Doppler spectra HH and VV are similar, which is expected because the radar resolution volumes are the same. A comparison between all the retrievals obtained from HH and VV Doppler spectra shows comparable results when the retrievals are height-smoothed \cite{9}.

In Fig. 1, the full width of the Doppler spectra is about 6 m s\(^{-1}\) but the range of Doppler velocities clearly differs of 2 m s\(^{-1}\) for OB1-MB and -3 m s\(^{-1}\) for OB1-OB2 due to the radial component of the wind (-\(v_0\)). The retrieved (-\(v_0\)) difference is 1.66 (1.27) m s\(^{-1}\) for OB1-MB and -2.41 m s\(^{-1}\) for OB1-OB2. In this example, the retrieval of the radial wind is slightly underestimated. Let us consider this issue in more details.
4. MEAN HORIZONTAL WIND CORRECTION

From the mean Doppler velocity profiles of the 3 beams of TARA, the horizontal wind (speed and direction) and the vertical Doppler velocity can be derived with a time resolution, which is less than 3 s. At this high time resolution, the horizontal wind estimation is prone to errors caused by the non-homogeneity of the precipitation and turbulence. The mean horizontal wind is thus calculated after averaging the Doppler velocity profiles during about 10 min to reduce those errors. Fig. 2 shows the expected mean horizontal wind contribution for the line-of-sights of beams MB and OB2, which is compared with the retrieval ($-v_0$) averaged on 3.5 min. Stratiform light rain was measured. The trend of the retrieval is fairly good. However, it is underestimated in magnitude. Consequently, we expect that $D_0$ will be underestimated for MB and overestimated for OB2. That is the case in Fig. 3. If the contribution of the 10.5 min averaged horizontal wind radial component is subtracted to the Doppler velocities of the Doppler spectrum before the retrieval procedure, the retrieval of $D_0$ may be improved. This is demonstrated in Fig. 4, where the agreement of the multi-beam retrieval $D_0$ is much better. $N_w$ is depicted as well, with (Fig. 6) and without mean horizontal wind correction (Fig. 5). The multi-beam intercept parameter is also significantly improved in Fig. 6.

5. ALGORITHM APPLICATION ON AVERAGED DOPPLER SPECTRA

The retrieval algorithm is carried out on averaged Doppler spectra for two reasons. The first one is to investigate whether there is agreement between the retrievals of averaged and non-averaged spectra.
precipitation, the high time resolution of less than 3 s may be used.

![Figure 7](image)

Figure 7. Four consecutive profiles of mean volume diameter (time resolution is 17.52 s). For each plot, the thick lines, blue and red, represent the profiles obtained from VV- and HH- 6-averaged Doppler spectra. Because the same radar resolution volume is probed, they have to be the same. The other 6 thin line profiles correspond to the D₀ retrieval based on the non-averaged VV-Doppler spectra.

Fig. 7 shows four comparisons between retrievals of D₀ based on averaged (thick line blue) and non-averaged Doppler spectra (thin lines). The agreement is good and the deviations may be of the order of 0.1 mm, which is consistent with the error analysis performed in [9]. The mean horizontal wind correction has been applied on all the Doppler spectra.

For the number of averages, a trade-off has to be made. On one hand, the number of averages should be small in order to measure non-homogeneous precipitation, which means different raindrop size distributions in the three looking directions of the radar. On the other hand, the number of averages should be large enough to decrease the impact of statistical fluctuations of the Doppler spectrum on the retrievals. The processing of TARA radar is based on two-averaged Doppler spectra. Further, we choose to average 6 of those Doppler spectra, which are first shifted to have the same mean Doppler velocity to reduce unwanted broadening due the radial wind component variability. The averaging procedure is thus based on 12 Doppler spectra. This choice is made based on [10] where, after 10 averages of the Doppler spectra, a bias of 0.1 dB for the spectral differential reflectivity, which is used for mixed-phase and ice cloud retrievals [5], is obtained.

### 6. PRELIMINARY CONCLUSIONS

When the differential reflectivity cannot be used because of single-polarized beam, near-vertical profiling or very light precipitation, the raindrop size distribution results obtained from the measurement of Doppler spectra show that two many variables need to be retrieved (D₀, Nw, μ, ν₀, σ₀). With the proposed retrieval technique, the correct estimation of D₀ is crucial. A good candidate to replace Z₀ is the radial component of the mean horizontal wind. With this knowledge, the Doppler spectra can be compensated for the horizontal wind influence. This strongly improves the estimation of D₀. Consequently the other parameters to retrieve are directly improved as well. This was shown for Nw.

### REFERENCES


