THE EFFECT OF DROPLET CLUSTERING ON THE STATISTICS OF RADAR BACKSCATTER FROM WATER CLOUDS

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ABSTRACT

The applicability of the basic concepts of radar theory to the scattering of water clouds is discussed in view of the recently observed radar discrepancy. Here, the applicability of the incoherent Rayleigh scattering assumption is questioned in the case of scattering from water clouds. The assumption that droplets move fast enough to change the inter-particle distance by more than a wavelength might be violated. A possible reason for the violation of this assumption is the formation of clusters inside the cloud volume, which behave as coherent structures. The degree of droplets' clustering can be derived by the distribution of power after coherent summation of backscattered electric fields. The computational tests show that there is consistency with the n-step isotropic Pearson's random walk over the two-dimensional phase space.

Key words: droplet clustering; radar discrepancy; Pearson's random walk; coherent radar backscatter; water clouds.

1. RADAR DISCREPANCY

It has been observed that low-level liquid water clouds are often invisible by cloud radars [1], which means that the millimeter-wave cloud radars operating at $35GHz$ and $94GHz$ are not always sensitivity enough to measure these clouds. The so-called radar discrepancy term refers to the deviation between the value of radar reflectivity factor estimated by radar measurements and the theoretical one predicted by the standard radar scattering theory. Particularly, in a study of radar backscattering by stratocumulus [2], for a water cloud with expected mean value of $-26dBZ$ radar reflectivity factor, the actual, measured by both $35GHz$ and $94GHz$ radars, mean value of radar reflectivity factor is $\sim -39dBZ$, which makes the radar discrepancy equal to $-13dBZ$.

1.1. Possible reasoning

The reason for this observed discrepancy is that, in most cases, the incoherent Rayleigh scattering assumption may not be valid for water clouds. In water clouds, the concentration of particles is of the order of several hundred droplets per cm^3, which denotes that if the cloud radar operational wavelength is of the mm order, interparticle distance between droplets is less than a wavelength. The last statement implies that the backscattered electric fields phase difference uniform distribution assumption is violated and, indeed, randomness condition required for incoherent scattering theory is not satisfied.

The classical radar theory only considers incoherent backscattered power by liquid water droplets. However, recently, cloud researchers are interested in the concept of "coherence". According to the definition of Battan [3], a target consisting of many scattering centers, which can move relatively fast with respect to each other, is incoherent. This is true for rain. However, it does not apply for a liquid water cloud since its droplets may not produce incoherent radar returns. Nevertheless, since atmospheric turbulence in the cloud initiates particles' clustering, the positions of cloud particles cannot be considered as perfectly random. The randomness of particles' position ensures the validity of the assumption for uniform distribution of the phase difference of backscattered electric fields due to the individual scatterers. The nonperfect randomness implies spatially correlated particles and, hence a coherent backscatter contribution to the total backscattered power [4]. The validity of the incoherent scattering assumption may be violated in turbulent conditions.

Together with the uniform distribution assumption, the second requirement for incoherent summation of the individual radar backscattered power indicates that droplets inside the illuminated cloud volume move fast enough to change the inter-particle distance by more than a wavelength. The latter requirement might also be violated since external forces, such as turbulence, may not result in randomized cloud droplet velocities. The major effect of turbulence is the clustering effect, which means that in the presence of turbulence cloud droplets form small clusters. These clusters behave as coherent

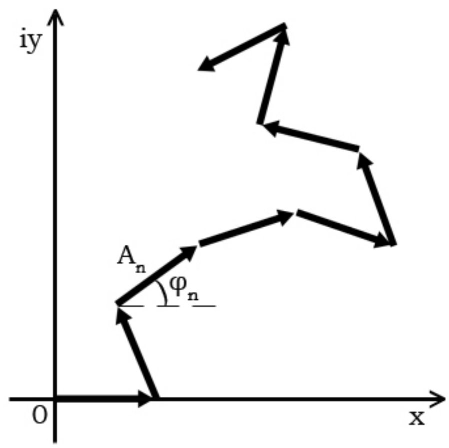


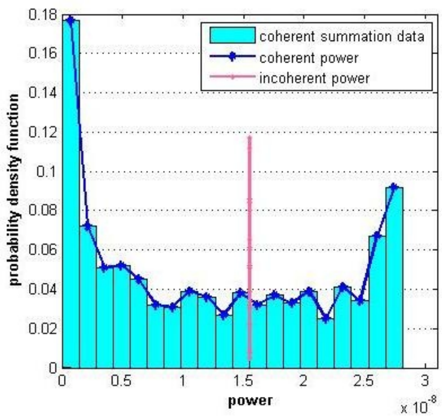
Figure 1: *A random walk in two-dimensional phase plane. The amplitude $A_{\rm n}$ denotes the n^{th} step of the random walk and the phase ϕ_n denotes the n^{th} step direction of the walk. All step directions are equally probable.*

structures because the position of their particles do not change randomly enough so as to result in uniformly distributed phase difference of the electric fields backscattered by the individual droplets [2]. Therefore, the classic radar theory demonstrating incoherent summation of radar backscattered powers should be questioned for the case of water clouds.

2. CLOUD DROPLETS' CLUSTERING

The backscattered electric field which is induced by random media is a random function of scatterer position. Since clouds are considered as random media, the propagation and scattering of electromagnetic radiation through clouds should be linked to the spatial and temporal distribution of droplets.

The spatial distribution of droplets in a cloud can be statistically characterized. Assume that the number of droplets inside the cloud volume is a random variable, but still countable. Then, the perfect randomness for a collection of particles can be considered as a statistically homogeneous Poisson process where the position of particles is uniformly distributed in the cloud volume and completely uncorrelated (i.e., statistically independent). On the other hand, in case of non-statistically independent particle positions (i.e., turbulent clustering), the random process is still statistically homogeneous but not spatially uncorrelated any more. Statistical homogeneity of this random process denotes that the mean particle density is spatially independent and non-deterministic [5]. As a result, in case of turbulent clustering, the randomness is not perfect since the positions of particles are correlated. Thus, it is obvious that spatial correlation implies a deviation from perfect randomness. Suppose N denotes the total number of clusters present inside the cloud volume.



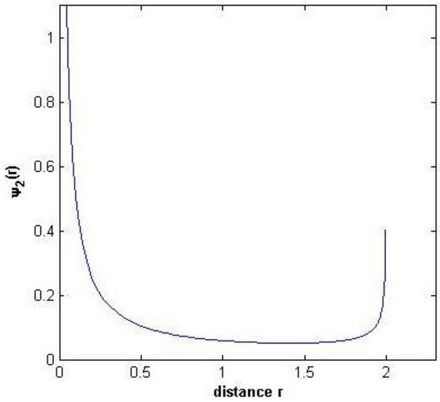


Figure 2: *Partially correlated droplets form 2 clusters, which consist of 31250 droplets each. In the upper graph, the PDF of power resulting from the coherent summation of backscatter electric fields is compared to the power from incoherent summation. The lower graph represents the PDF of the distance from the origin in a two-step isotropic Pearson's random walk over the two-dimensional phase space.*

Droplets' clustering has been investigated in [6] through three cases:

- N particles totally correlated form one cluster.
- N particles partially correlated form n=2,3,. . . , N-1 clusters.
- N particles completely uncorrelated do not form any cluster.

Clusters have been considered in alignment to X-axis, Y-axis, Z-axis or randomly arranged in the cloud volume. The cloud volume is simulated as a box of 0.05 x 0.05 x 0.05 m dimensions and the concentration of droplets in the volume is 500 droplets/cm^3, which means that the total number of droplets is N=62500. The turbulence model allows the clusters to shift either horizontally, vertically, towards any direction in 3-dimensional space or rotate about a vertical central axis. The induced velocity and acceleration define the maximum displacement of

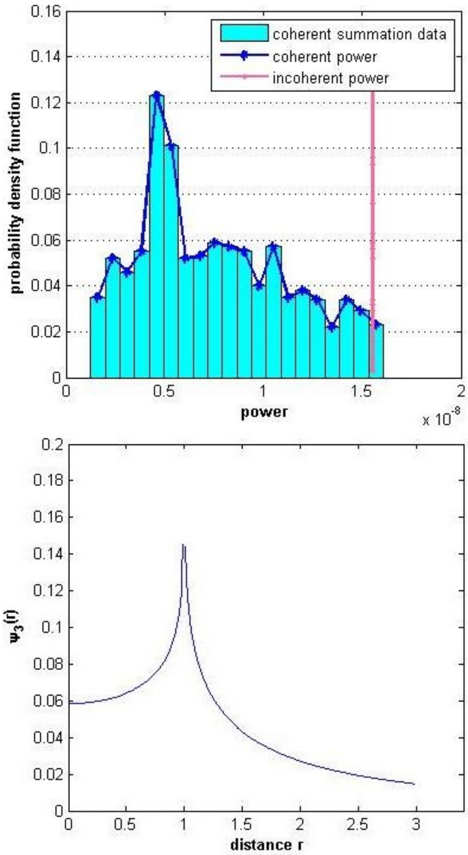


Figure 3: *Partially correlated droplets form 3 clusters, which consist of 20833 droplets each. In the upper graph, the PDF of power resulting from the coherent summation of backscatter electric field is compared to the power from incoherent summation. The lower graph represents the PDF of the distance from the origin in a three-step isotropic Pearson's random walk over the two-dimensional phase space.*

clusters movement. The operational radar frequency is $94GHz$. The total measurement time is 1sec and radar collects a measurement every 1ms. Hence, 1000 measurements have been averaged in the following results.

3. RANDOM WALKS APPROACH

The statistical properties of electromagnetic waves backscattered by objects containing few scattering centers can be studied through random walks theory. In Fig.1, a random walk in the complex plane is illustrated. Suppose radar is located at a distance $\mathbf{x}_{\mathrm{R}}$ from a given reference system. Each n^{th} cloud droplet is located at a distance $\mathbf{x}_{\mathrm{n}}$, $n = 1, 2, \ldots, N$ respectively and contributes to the total backscattered electric field. The total backscattered electric field measured at the position of the radar is given by:

$$\mathbf{E}^{\mathrm{sc}}(\mathbf{x}_{\mathrm{R}}) = \sum_{\mathrm{k}=1}^{3} \mathbf{E}_{k}^{\mathrm{sc}}(\mathbf{x}_{\mathrm{R}}) = \sum_{\mathrm{k}=1}^{3} \sum_{\mathrm{n}=1}^{N} \mathbf{E}_{k}^{\mathrm{sc}}(\mathbf{x}_{\mathrm{R}}, \mathbf{x}_{\mathrm{n}}), \quad (1)$$

where the k^{th} component of electric field backscattered by the n^{th} individual droplet is $\mathbf{E}_{k}^{\mathrm{sc}}(\mathbf{x}_{\mathrm{R}}, \mathbf{x}_{\mathrm{n}}) = A_{\mathrm{n}} e^{i\phi_n}$. The magnitude A_{n} and phase ϕ_n of backscattered electric field denotes the n^{th} step length and step direction of the random walk respectively. All step directions are equally probable. Each backscattered electric field $\mathbf{E}_{k}^{\mathrm{sc}}(\mathbf{x}_{\mathrm{R}}, \mathbf{x}_{\mathrm{n}})$ is represented by a vector in the complex plane. Thus, the coherent summation of the individual backscattered electric fields constitute a *random walk* in the complex plane.

4. PRIMARILY RESULTS

In Fig. 1, the summation of N vectors in the complex plane represents the k^{th} component of total electric field backscattered by the total N droplets present in the cloud volume. Since cloud is a random medium, this vector sum is also random [7]. The statistical properties of k^{th} component of the total backscattered electric field and the corresponding total backscattered power were extracted for different degrees of spatial correlation among droplets. In particular, the impact of several degrees of droplet's clustering (i.e., the number of clusters present in the cloud volume) on backscattered power distribution was investigated.

The computational outcomes confirm that the probability density function (PDF) of the power resulting from the coherent summation of backscatter electric field is consistent with the probability density function of the distance from the origin in an n-step isotropic Pearson's random walk over the two-dimensional phase space. In case of N totally correlated particles, which form one single cluster, the power distribution is a delta function which is consistent with a single-step random walk. This is a trivial case, since after one step in the plane, walker's position will lie on a ring around the initial position. In [6], the electric field representation in complex plane is illustrated as a ring around the origin $(0, 0)$, in case all cloud droplets belong to a single cluster. That means that the PDF of the power estimated from the coherent summation of backscatter electric fields is represented by a dirac function.

The PDF of Pearson's two-step walk $\psi_2(r)$ with constant step length $\alpha = 1$ is shown in the lower graph of Fig. 2 [8]. By comparing the upper and lower graph of Fig. 2, the consistency between the distribution of power backscattered by 2 clusters, consisting of N=31250 cloud droplets each, and the PDF of two-step random walk is validated. The PDF of Pearson's three-step walk $\psi_3(r)$ with constant step length $\alpha = 1$ is shown in the lower graph of Fig. 3 [8]. By comparing the upper and lower graph of Fig. 3, the consistency between the distribution

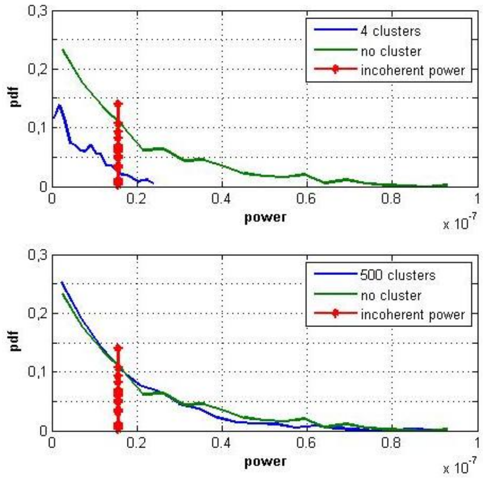


Figure 4: *Partially correlated droplets form more clusters. In the upper graph, the PDF of power resulting from the coherent summation of backscatter electric fields, when particles are split into 4 clusters, differs from the exponential distribution of non-correlation. In the lower graph, the PDF of power resulting from the coherent summation of backscatter electric fields, when particles are split into 500 clusters, converges to the exponential distribution of non-correlation.*

of power backscattered by 3 clusters of N=20833 cloud droplets each and the PDF of three-step random walk is validated.

Especially from the computational tests performed for the cases of two and three clusters, discrepancies between the mean power, resulting from the coherent summation of electric fields, and the expected power, estimated from incoherent summation of electric fields, were observed. In detail, for the case n=3 clusters, which are arranged in alignment with X-axis inside the cloud volume and move towards any random direction in 3-Dimensional space within a maximum displacement of $150\mu m$, the PDF of backscattered power is illustrated in the upper graph of Fig. 3. For these specific initial conditions, mean value of backscattered power was approximately 50% lower than the expected incoherent power. Thus, it is likely that a ground-based radar is not able to detect such a cloud volume filled with spatial correlated droplets.

For the case of n=2 clusters, which are randomly arranged inside the volume and move towards any random direction in 3-Dimensional space within a maximum displacement of $150\mu m$, PDF of backscattered power is shown in the upper graph of Fig. 2. From the results of this computational test, discrepancy between the mean value of backscattered coherent power and incoherent power is low. However, from the shape of PDF in Fig. 2, there is still probability that a ground-based cloud radar is not able to detect these correlated cloud droplets.

Other degrees of correlation (corresponding to more clusters) are investigated until the power PDF converges to the exponential distribution of N completely uncorrelated particles. This is illustrated in Fig.4 where the power distribution for the case of 4 clusters, consisting of 15625 cloud droplets each, deviates from the non-correlation case. However, 500 clusters, consisting of 125 cloud droplets each, result in power distribution illustrated in the lower graph of Fig. 4, where it is evident that power distribution converges to the exponential distribution of uncorrelated droplets. Convergence had been observed already from the case of 10 clusters present in the cloud volume. The simulations showed that convergence may appear at a lower or higher degree of clustering (i.e., 9 or 13 clusters), depending on the initial conditions.

5. CONCLUSIONS

The clustering of particles inside the cloud volume results in a radar response deviating from the one predicted by the standard radar theory. For high degree of spatial correlation among droplets, such as 2 or 3 clusters present in the cloud volume, the shape of PDF of backscattered power is characteristic and consistent with the analytical expressions from Pearson's 2- or 3-step random walk. Not only the PDF of the power backscattered by partially correlated particles differs from the well-known exponential distribution, but there are systematic discrepancies in the mean (expected) power as well. Therefore, the radar reflectivity sensitivity of a ground-based cloud radar may not be sufficient for detecting backscattered signals from water clouds.

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