Surface Runoff

As for many other hydrological models (Singh and Frevert, 2002) surface routing is calculated according to the kinematic wave approximation of the shallow water equation (Lighthill and Whitam, 1955). Overland surface routing and channel flow are described by the continuity and momentum conservation equations:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial t} = q$$

$$Q = \alpha A^{m}$$

where A is the flow cross-sectional area, Q is the flow rate of water discharge, q is the rate of lateral inflow per unit of length due to all the physical processes contributing to the hydrological cycle, t is time, x is the coordinate along the river path, α is the kinematic wave parameter, and m the kinematic wave exponent usually assumed = 1. The kinematic wave parameter α has the dimension of speed and it can be written as:

$$\alpha = \frac{S^{1/2} R^{2/3}}{n(\mu)}$$

where S the longitudinal bed slope of the flow element, n is the Manning's roughness coefficient depending on the land use type μ, R is the hydraulic radius that can be calculated as a linear function of the drained area D according to:

$$R = \beta + \gamma D^{\delta}$$

where β, γ and δ are empirical constants to tune with during the calibration. The exponent δ is usually very close to 1.

Evapotranspiration

The potential evapotranspiration is computed as a function of the evapotranspiration in saturated soil conditions (Thornthwaite and Mather, 1957), according to the formula:

$$\mathbf{ET_p = k_c ET_0}$$

where k_c is the crop factor that is a function of crop type. The reference evapotranspiration ET_0 is approximated as a linear function of temperature and is calculated according to:

$$\mathbf{ET_0 = \alpha + \beta N W_{ta}(h, T) T}$$

where N is daily maximum sunshine hours, $W_{ta}(h,T)$ is the compensation factor depending on the elevation h and temperature T. The coefficients α and β are to be estimated and this is carried out by fitting with the least squared method the Thornthwaite formula, namely:

$$\mathbf{16 \frac{n(m)}{30} \frac{N(m)}{12} \left[10 \frac{T(m)}{K_1} \right]^{K_2} = \alpha + \beta N W_{ta}(h, T) T}$$

where $n(m)$ is the number of days of month m, N is the daily maximum sunshine hours for the month m, $T(m)$ is the monthly average temperature, k_1 and k_2 are the thermal indexes.

The compensation factor $W_{ta}(h,T)$ is a function of elevation and temperature and is calculated from:

$$W_{ta}(h,T) = A(h)T^2 + B(h)T + C(h)$$

The coefficient $A(h)$, $B(h)$ and $C(h)$ have been estimated for different range of elevation according to the table reported by Dorembos et al. (1984).

The actual evapotranspiration ET_A is a fraction of the potential evapotranspiration ET_p and it is calculated as a linear function of ground relative humidity G_{RH}, more specifically ET_A is zero in arid condition (G_{RH}<0.2) and it is equal to ET_p for G_{RH}>0.7. For other values of ground humidity, the evapotranspiration term is calculated as a linear function of G_{RH}:

$$ET_A = \frac{G_{RH} - 0.2}{0.7 - 02} ET_P = \frac{G_{RH} - 0.2}{0.7 - 02} k_c \cdot ET_0$$

Additional details on the estimation of the evapotranspiration term can be found in Todini (1996) and Thornthwaite and Mather (1955).

Melting

Chym incorporates a temperature-index melt model based on the assumption that the melt rate is the sum of two terms. The first is linearly related to air temperature, which is regarded as an integrated index of the total energy available for melting. The second term is proportional to the incoming net solar radiation. Within this approach, melting is assumed to occur when the temperature T is above a threshold level *TT* (typically 1°C). It has been recognized (Pellicciotti et al., 2005) that this approach reproduce in a realistic way the observed melting rate in the Alpine region. The melting rate *M* (mm of equivalent precipitation per hour) is calculated as:

$$M = T_F T + S_{RF}(1-\alpha)G_{\downarrow}$$

The factor of proportionality for the first term T_F is the so called temperature factor (typical value around 0.05 mm/°C), the coefficient S_{RF} is the shortwave radiation factor, and its typical value is around 0.0094 mm/h M^2/(Watt °C), α is the fraction of solar radiation reflected by the surface, T is the ground temperature estimated by CHyM. In the previous formula $G_{\downarrow}$ is the incoming short wave solar radiation estimated from:

$$G_{\downarrow} = C_s A_{tr} \sin(\Psi)$$

where C_s is the solar constant (1368 Watt/m2) and A_{tr} is the net sky trasmissivity that can be approximated by (Stull, 1999):

$$A_{tr} = \left[0.6 + 0.2\sin(\Psi)\right](1.0 - 0.4\sigma_H)(1.0 - 0.7\sigma_M)(1.0 - 0.4\sigma_L)$$

where σ_H, σ_M and σ_L are the fractions of cloud cover respectively for high, medium and lower levels and their values are assumed to be a simple linear function of the time of year. The sinusoidal function of the solar elevation angle

$\sin(\Psi)$ depends on the latitude φ and longitude λ of the location and it is calculated as follow:

$$\sin(\Psi) = \sin(\varphi)\sin(\delta_s) - \cos(\varphi)\cos(\delta_s)\cos(\frac{2\pi t_{utc}}{t_d} - \lambda)$$

where t_{utc} is the time of the day in Universal Time Coordinate (also known as Greenwich Mean Time or GMT), t_d is the length of the day and Ψ is the solar declination angle. For practical purposes, the second term of the melting contribution is considered only if the angle Ψ is in the interval $0 < \Psi \geq \pi / 2$, namely during the daytime. The values of the temperature factor T_F and the shortwave radiation factor S_{RF} are calibrated by "train and error".

Infiltration ad Percolation

The infiltration process is modelled using a conceptual model similar to those proposed by several authors (e.g. Overton 1964; Singh and Yu 1990). Within this approach we describe the soil as composed of two reservoirs of water: the precipitation infiltrates in the upper soil layer until the saturation level is reached. The water of the upper layer also infiltrate (percolation) toward the lower soil layer. The total amount of water that infiltrates, I, is also saved at each time step in order to evaluate the return flow (see below).

Return flow

The return flow is parameterized assuming that the contribution to each elementary channel-cell is proportional to the total infiltration in the upstream basin during the last N months

$$\mathbf{R_f} = \int_{\mathbf{Up}} \mathbf{ds} \int \mathbf{I}\,(\mathbf{t}, \mathbf{s})\mathbf{dt}$$

the infiltration term *I* described in the previous paragraph is integrated over the whole upstream basin of each cell and the time integral is carried out over the last *N* months, *N* being a value to be optimized during the calibration process. In practice, we assume that infltrated water contributes to the return flow of the same basin. The return flow is calculated for each cell as a linear function of the R_f term, and the linear coefficient is optimized during the calibration phase with typical values around $5x10^{-7}$ mm $hour^{-1}$ Km^{-2}.